Form Factors for Exclusive Semileptonic B–Decays

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Abstract

We developed the new parton model approach for exclusive semileptonic decays of B-meson to D, D^* by extending the inclusive parton model, and by combining with the results of the HQET, motivated by Drell-Yan process. Without the nearest pole dominance ansätze, we **derived** the dependences of hadronic form factors on q^2. We also calculated numerically the slope of the Isgur-Wise function, which is consistent with the experimental results.

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I. INTRODUCTION

In exclusive weak decay processes of hadrons, the effects of strong interaction are encoded in hadronic form factors. These decay form factors are Lorentz invariant functions which depend on the momentum transfer q^2, and their behaviors with varying q^2 are dominated by non-perturbative effects of QCD.

B–meson decay processes have been studied in detail as providing many interesting informations on the interplay of electroweak and strong interactions and as a source extracting the parameters of weak interactions, such as $|V_{cb}|$ and $|V_{ub}|$. As more data will be accumulated from the asymmetric B–factories in near future, the theoretical and experimental studies on exclusive B–meson decays would also give better understandings on the Standard Model and its possible extensions.

Over the past few years, a great progress has been achieved in our understanding of the exclusive semileptonic decays of heavy flavors to heavy flavors [1]. In the limit where the mass of the heavy quark is taken to infinity, its stong interactions become independent of its mass and spin, and depend only on its velocity. This provides a new $SU(2N_f)$ spin–flavor symmetry, which is not manifest in the theory of QCD. However, this new symmetry has been made explicit in a framework of the heavy quark effective theory (HQET) [2]. In practice, the HQET and this new symmetry relate all the hadronic matrix elements of $B \to D$ and $B \to D^*$ semileptonic decays, and all the form factors can be reduced to a single universal function, the so-called Isgur-Wise function [2,3], which represents the common non-perturbative dynamics of weak decays of heavy mesons. However, the HQET cannot predict the values of the Isgur-Wise function over the whole q^2 range, though the normalization of the Isgur-Wise function is precisely known in the zero recoil limit. Hence the extrapolation of q^2 dependences of the Isgur-Wise function and of all form factors is still model dependent and the source of uncertainties in any theoretical model. Therefore, it is strongly recommended to determine hadronic form factors of B–meson decay more reliably, when we think of their importance in theoretical and experimental analyses.

In this paper we developed the parton model approach **for exclusive semileptonic** B decays to D, D^*, and predicted the q^2 dependences of all form factors. Previously the parton model approach has been established to describe inclusive semileptonic B decays [4,5], and found to give excellent agreements with experiments for electron energy spectrum at all energies. While many attempts describing exclusive B decays often take the pole-dominance ansätze as behaviors of form factors with varying q^2 [6,7], in our approach they are derived by the kinematical relations between initial b quark and final c quark. According to the Wirbel *et. al.* model [6], which is one of the most popular model to describe exclusive decays of B mesons, the hadronic form factors are related to the meson wavefunctions' overlap-integral in the infinite momentum frame, but in our model they are determined by integral of the fragmentation functions, which are experimentally measuable.

For completeness, here we briefly review the parton model approach for inclusive semileptonic decays of B meson. The parton model approach pictures the mesonic decay as the decay of the partons in analogy to deep inelastic scattering process. The probability of finding a b-quark in a B meson carrying a fraction x of the meson momentum in the infinite momentum frame is given by the distribution function $f(x)$. Then we write the Lorentz invariant decay width as follows:

$$ E_B \, d\Gamma(B \to X_q e\nu) = \int \, dx \, f(x) \, E_b \, d\Gamma(b \to qe\nu) \;\;, \tag{1} $$

with the relation $p_b = xp_B$. Using standard definitions of the structure functions for the hadronic tensor $W^{\mu\nu}$

$$ W^{\mu\nu} = -g^{\mu\nu} \, W_1 + p_B^\mu p_B^\nu \, \frac{W_2}{m_B^2} - i\epsilon^{\mu\nu\alpha\beta} p_{B\alpha} q_\beta \, \frac{W_3}{2m_B^2} $$
$$ + q^\mu q^\nu \, \frac{W_4}{m_B^2} + (p_B^\mu q^\nu + p_B^\nu q^\mu) \, \frac{W_5}{2m_B^2} + i(p_B^\mu q^\nu - p_B^\nu q^\mu) \, \frac{W_6}{2m_B^2} \;\;, \tag{2} $$

we obtain

$$ W_1 = \frac{1}{2} \left(f(x_+) + f(x_-) \right) \;\;, $$
$$ \frac{W_2}{m_B^2} = \frac{2}{m_B^2(x_+ - x_-)} \left(x_+ f(x_+) - x_- f(x_-) \right) \;\;, $$

$$\frac{W_3}{2m_B^2} = -\frac{1}{m_B^2(x_+ - x_-)}\left(f(x_+) - f(x_-)\right) \quad,$$
$$\text{and} \quad W_4 = W_5 = W_6 = 0 \quad,$$

where

$$x_\pm = \frac{q_0 \pm \sqrt{|\mathbf{q}|^2 + m_q^2}}{m_B} \quad,$$

with the final state quark mass m_q. Hence the double differential decay rate is given by

$$\frac{d\Gamma}{dE_e dy dx_+} = \frac{G_F^2 M_B^4 |V_{qb}|^2}{8\pi^3} y\{x_+ f(x_+)(x_+ - y - m_q^2/m_B^2/x_+) - (x_+ \leftrightarrow x_-)\} \quad, \tag{3}$$

where $y = 2E_\nu/m_B$. In the limit $f(z) = \delta(1 - z)$, we reproduce the HQET leading term except that m_b is replaced by m_B. In Section II, we develop the parton model approach for exclusive semileptonic decays of B meson, and give all the theoretical details for $B \to Dl\nu$ and $B \to D^*l\nu$. Section III contains discussions and conclusions of this paper.

II. PARTON MODEL APPROACH FOR EXCLUSIVE DECAYS OF B MESON

We now develop the parton model approach for exclusive semileptonic decays of B meson by extending the previously explained inclusive parton model, and by combining with the results of the HQET. Theoretical formulation of this approach is, in a sence, closely related to Drell-Yan process, while the parton model of inclusive B decays is motivated by deep inelastic scattering process. And the bound state effects of exclusive B decays are encoded into the hadronic distribution functions of partons inside an initial B meson and of partons of a final state resonance hadron. Then, the Lorentz invariant hadronic decay width can be obtained using the structure functions, as in Eq. (1),

$$E_B \cdot d\Gamma(B \to D(D^*)e\nu) = \int dx \int dy \, f_B(x) \, E_b \cdot d\Gamma(b \to ce\nu) \, f_D(y) \quad. \tag{4}$$

The first integral represents the effects of motion of b quark within B meson and the second integral those of c quark within D meson. The variables x and y are fractions of momenta of partons to momenta of mesons,

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$$p_b = x p_B \ , \qquad p_c = y p_D \ , \tag{5}$$

in the infinite momentum frame. The functions $f_B(x)$ and $f_D(y)$ are the distribution func-
tion of b quark inside B meson, and the fragmentation function of c quark to D meson
respectively. Since the momentum fractions and the distribution functions are all defined in
the infinite momentum frame, we have to consider the Lorentz invariant quantity, $E \cdot d\Gamma$ as
defined in Eq. (1) and (4), to use at any other frame.

For a heavy quark $(Q = t, b, c)$ the distribution and fragmentation functions in a heavy
meson $(Q\bar{q})$, which are closely related by a time reversal transformation, are of similar
functional forms, and peak both at large value of x. Therefore, we follow the previous work
of Paschos $et.$ $al.$ [4] to use the Peterson's fragmentation function [9] for both distributions,
$f_B(x)$ and $f_D(y)$. It has the functional form:

$$f_Q(z) = N z^{-1} \left(1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right)^{-2} \ , \tag{6}$$

where N is a normalization constant, and Q denotes b or c quark.

In the Drell-Yan process, the rest degrees of freedom of initial nucleons which do not
take part in the scattering make incoherent final states, see in Fig. 1 (a). In the exclusive
semileptonic decay of a heavy meson into a final state heavy meson, however, two sets of
left-over light-degrees of freedom are summed to have the connection,

$$|< D(\text{or } D^*)|J|B >|^2 \sim \sum_{\text{spin}, X_1, X_2} | \{< X_2| < c|\} \, J \, \{|b > |X_1 >\} \, |^2 \ . \tag{7}$$

For more explicit meaning of Eq. (7), see Fig. 1 (b) and Section II A and B. Two sets of
states, $|X_1 >$, $|X_2 >$ are not independent here, in fact. And to connect them we need a
relation between x and y from the decay kinematics. The momentum transfer of the decay
between mesons is defined as

$$q \equiv p_B - p_D \ . \tag{8}$$

On the other hand, the momentum transfer of the partonic subprocess is given by

$$q^{(\text{parton})} = p_b - p_c = x p_B - y p_D \ . \tag{9}$$

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In fact, the heavy meson's momentum would be $p_H = p_Q + k + \mathcal{O}(1/m_Q)$, where $H = B, D$ or D^*, and $Q = b$ or c. And k denotes the momentum of the light-degrees of freedom, and is related to the effective mass of a common light quark, $\bar{\Lambda}$. Therefore we have $q = q^{(\text{parton})}$ up to the common part of the $1/m_Q$ corrections. With these kinematic relations we derived the following relation

$$y(x, q^2) = \frac{1}{m_D}\sqrt{x(x-1)m_B^2 + (1-x)q^2 + xm_D^2} \quad . \tag{10}$$

Substituting y of Eq. (4) for $y(x)$ of Eq. (10), the double integral of Eq. (4) is reduced to the single integral over x. Using this relation, we can sum the intermediate states in Fig. 1(b), as in (7). In Fig. 1, we show the schematic diagrams of Drell-Yan process and the related exlusive semileptonic decay of B meson.

We note here that the connection (7) and the kinematic relation (10) are valid approximations for the heavy-to-heavy resonance decays, with the common light-degrees of freedom of the size $\mathcal{O}(1/m_Q)$. As explained before, in the limit where $f_Q(x) = \delta(1-x)$ by increasing m_Q to infinity, we can reproduce the HQET leading term. By comparison, the inclusive parton model approach is more reliable for the heavy-to-light non-resonant decays to final states of many particles.

A. $B \to De\nu$

From Lorentz invariance we write the matrix element of the decay $\bar{B} \to De\bar{\nu}$ in the form

$$< D|J_\mu|B> = f_+(q^2)(p_B + p_D)_\mu + f_-(q^2)(p_B - p_D)_\mu \quad , \tag{11}$$

and in terms of the HQET

$$< D(v')|J_\mu|B(v)> = \sqrt{m_B m_D}\,(\xi_+(v \cdot v')(v + v')_\mu + \xi_-(v \cdot v')(v - v')_\mu) \quad . \tag{12}$$

Due to the conservation of leptonic currents, the form factors multiplicated by q_μ do not contribute. Then the hadronic tensor is given by

$$H_{\mu\nu} = < D|J_\mu|B > < D|J_\nu|B >^*$$

$$= 2 \, |f_+(q^2)|^2 (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \quad , \tag{13}$$

and can be expressed by the Isgur-Wise function,

$$H_{\mu\nu} = R^{-1} |\xi(v \cdot v')|^2 (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \left(1 + \mathcal{O}(\frac{1}{m_Q})\right) \quad , \tag{14}$$

where

$$R = \frac{2\sqrt{m_B m_D}}{m_B + m_D} \quad .$$

The partonic subprocess decay width is given by

$$E_b \cdot d\Gamma(b \to ce\nu) = \frac{1}{2}(2\pi)^4 \delta^4(p_b - p_c - q) \cdot 2 G_F^{\,2} |V_{cb}|^2 H^{(\text{parton})}_{\mu\nu} L^{\mu\nu}$$

$$\times \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} \quad , \tag{15}$$

where

$$L^{\mu\nu} = 2 \; (p_e^\mu p_\nu^\nu + p_e^\nu p_\nu^\mu - g^{\mu\nu} p_e \cdot p_\nu + i\epsilon^{\mu\nu\alpha\beta} p_{e\alpha} p_{\nu\beta}) \quad ,$$

$$H^{(\text{parton})}_{\mu\nu} = N \; (p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu}) \quad . \tag{16}$$

Here $H^{(\text{parton})}_{\mu\nu}$ denotes the partonic subprocess' hadronic tensor contrbuting $B \to D$ decay. In the inclusive decays $B \to X_c e\nu$, the hadronic tensor is given by

$$W_{\mu\nu} = 2 \; (p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu} - g_{\mu\nu} p_b \cdot p_c - i\epsilon_{\mu\nu\alpha\beta} p_b^\alpha p_c^\beta) \quad .$$

As can be easily seen, this hadronic tensor contains all the possible spin configurations of $b \to c$ transition from spin 0 state. Here we are interested in only $B \to D$ process, where the spin does not change during the process. Therefore, we have to choose only the spin-inert part which contributes to $B \to D$ process from the inclusive hadronic tensor, $W_{\mu\nu}$. By comparing with Eq. (14), we find that the spin-inert part has the form of $(p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu})$, as shown in (16). Besides, we do not know how much spin-inert part really contributes to $B \to D$ semileptonic decay, because other decays such as $B \to D^*$, $B \to D^{**}$ also contain

spin-inert parts. Therefore, the parameter N is introduced to estimate the size of spin-inert part which contributes to $B \rightarrow D$ process.

Generally we can write the hadronic tensor of any exclusive semileptonic decay modes for B mesons as

$$H_{\mu\nu}^{(\text{parton})} = H_1(q^2) \ (p_{b_\mu} p_{c_\nu} + p_{b_\nu} p_{c_\mu}) + H_2(q^2) \ g^{\mu\nu} p_b \cdot p_c + i H_3(q^2) \ \epsilon^{\mu\nu\alpha\beta} p_{b_\alpha} p_{c_\beta} \ , \tag{17}$$

which is expressed in the form motivated by that of inclusive $B \rightarrow X_c e\nu$ decays. With the expression of Eq. (17), we find

$$H_1 = N, \qquad H_2 = H_3 = 0.$$

in the case of $B \rightarrow De\nu$ decay. The constant N will be later determined by the zero recoil limit of the Isgur-Wise function.

Using the relation (5), we can write the hadronic tensor in the parton level as follows

$$H_{\mu\nu}^{(\text{parton})} = N \ xy \ (p_{B_\mu} p_{D_\nu} + p_{B_\nu} p_{D_\mu}) \ . \tag{18}$$

The momentum conservation of the partonic subprocess corresponds to the momentum conservation in the hadronic level in our model, as explained before. So we can substitute the Dirac delta function $\delta^4(p_b - p_c - q)$ for $\delta^4(p_B - p_D - q)$ in Eq. (15) with no loss of generality. Therefore, we write the decay width of $\bar{B} \rightarrow De\bar{\nu}$,

$$
\begin{aligned}
E_B \cdot d\Gamma(B \rightarrow De\nu) &= \int dx \ f_B(x) \ f_D(y(x,q^2)) E_b \ d\Gamma(b \rightarrow ce\nu) \\
&= \int dx \ f_B(x) f_D(y(x,q^2)) \ (2\pi)^4 \delta^4(p_B - p_D - q) \\
&\quad \times G_F^2 |V_{cb}|^2 \ N \ xy(x,q^2) \ (p_{B_\mu} p_{D_\nu} + p_{B_\nu} p_{D_\mu}) L^{\mu\nu} \\
&\quad \times \ y^2(x,q^2) \ \frac{d^3 p_D}{(2\pi)^3 2E_D} \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} \ .
\end{aligned} \tag{19}
$$

Hence we find the hadronic tensor

$$
\begin{aligned}
H_{\mu\nu}(B \rightarrow De\nu) &= N \int dx \ f_B(x) f_D(y(x,q^2)) \ xy^3(x,q^2) \ (p_{B_\mu} p_{D_\nu} + p_{B_\nu} p_{D_\mu}) \\
&\equiv N \ \mathcal{F}(q^2)(p_{B_\mu} p_{D_\nu} + p_{B_\nu} p_{D_\mu}) \ ,
\end{aligned} \tag{20}
$$

where we defined the function $\mathcal{F}(q^2)$ as

$$\mathcal{F}(q^2) \equiv \int dx \; f_B(x) f_D(y(x, q^2)) \; xy^3(x, q^2) \quad . \tag{21}$$

For given q^2 in our parton picture, the function $\mathcal{F}(q^2)$ measures the weighted transition amplitude, which is explicitly given by the overlap integral of distribution functions of initial and final state hadrons.

Comparing (20) with the Eq. (14), the Isgur-Wise function is calculated within the parton model approach

$$|\xi(v \cdot v')|^2 \left(1 + \mathcal{O}(\frac{1}{m_Q})\right) = 4N \cdot R \cdot \mathcal{F}(v \cdot v') \quad . \tag{22}$$

As explained before, in our model all the non-perturbative QCD effects are included through the distribution functions, so the function $\mathcal{F}(q^2)$ validly contains all $1/m_Q$ corrections. From the value of the zero recoil limit of the Isgur-Wise function with $1/m_Q$ corrections [8], we can determine the numerical value of N. In Table 1, we show the numerical values of N with varying the parameters of the fragmentation functions, (ϵ_b, ϵ_c). Unfortunately we cannot obtain the analytic form of $\mathcal{F}(q^2)$ due to the nontriviality of the integrals of $f(x)$. However, we can numerically obtain the behaviors of the Isgur-Wise function with varying q^2 from the Eq. (22), and calculate its slope parameter. The normalization constant N and the slope parameters ρ^2 with the input values of (ϵ_b, ϵ_c) are also shown in the Table 1. We varied ϵ_b from 0.004 to 0.006, and ϵ_c from 0.04 to 0.1.

The q^2 spectrum is given by

$$\frac{d\Gamma(\bar{B} \rightarrow De\bar{\nu})}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3 m_B^3} \mathcal{F}(q^2) \left((m_B^2 - m_D^2 + q^2)^2 - 4m_B^2 q^2\right)^{3/2} \quad . \tag{23}$$

And in Fig. 2(a), our prediction is shown with the parameters (ϵ_b =0.004, ϵ_c =0.04), compared with Wirbel $et\ al.$'s model prediction [6], which shows quite a good agreement.

B. $B \rightarrow D^* e\nu$

We write the matrix element of the decay $\bar{B} \rightarrow D^* e\bar{\nu}$ in familiar form

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$$< D^*|V_\mu + A_\mu|B > = \frac{2i}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_{D^*}^\alpha p_B^\beta V(q^2)$$

$$+ (m_B + m_{D^*}) \epsilon^*_\mu A_1(q^2) - \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} (p_B + p_{D^*})_\mu A_2(q^2)$$

$$- 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_3(q^2) + 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_0(q^2) \quad , \tag{24}$$

where

$$A_3(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(q^2) \quad ,$$

and in terms of the HQET

$$< D^*(v')|V_\mu + A_\mu|B(v) > = \sqrt{m_B m_{D^*}}$$

$$\times (i\xi_V(v \cdot v')\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^\alpha v^\beta v(q^2) + \xi_{A_1}(v \cdot v')(v \cdot v' + 1)\epsilon^*_\mu$$

$$- \xi_{A_2}(v \cdot v')\epsilon^* \cdot v v_\mu - \xi_{A_3}(v \cdot v')\epsilon^* \cdot v v'_\mu) \quad . \tag{25}$$

In fact, $A_0(q^2)$ and $A_3(q^2)$ do not contribute here because of leptonic currents conservation. With the heavy quark expansion, the hadronic form factors are related to the Isgur-Wise function $\xi(v \cdot v')$, such as $\xi_i(v \cdot v') = \xi(v \cdot v')(\alpha_i + \mathcal{O}(1/m_Q))$, where $\alpha_{A_2} = 0$ and $\alpha_i = 1$ otherwise.

The hadronic tensor of $\bar{B} \to D^* e \bar{\nu}$ decay is obtained using the HQET

$$H_{\mu\nu} = \sum_{\text{spin}} < D^*|V_\mu + A_\mu|B >< D^*|V_\nu + A_\nu|B >^* \tag{26}$$

$$= R^{*-1}|\xi(v \cdot v')|^2 \left[\left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) (p_{B\mu} p_{D^*\nu} + p_{B\nu} p_{D^*\mu})(1 + \mathcal{O}(\frac{1}{M_Q})) \right.$$

$$\left. - 2 \left(1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) \left(g_{\mu\nu} p_B \cdot p_{D^*}(1 + \mathcal{O}(\frac{1}{M_Q})) - i\epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D^*}^\beta(1 + \mathcal{O}(\frac{1}{M_Q})) \right) \right] \quad ,$$

where $R^* = 2\sqrt{m_B m_{D^*}}/(m_B + m_{D^*})$. Investigating the above relation, we can find the form of the hadronic tensor for the partonic subprocess which contributes to the decay $\bar{B} \to D^* e \bar{\nu}$. The resulting hadronic tensor is written down in the form,

$$H_{\mu\nu}^{(\text{parton})} = \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) N_1 \, (p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu})$$

$$- 2 \left(1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) (N_2 \, g_{\mu\nu} p_b \cdot p_c - iN_3 \, \epsilon_{\mu\nu\alpha\beta} p_b^\alpha p_c^\beta) \quad . \tag{27}$$

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The parameters N_i's give the relative size of form factors and overall normalization. In general they are not constants and have the q^2 dependences. In the heavy quark limit, N_i's become constants and the values are equal to that of the normalization constant N in $B \to De\nu$ process.

In order to investigate the procedure more conveniently, we define the ratios of form factors as follows:

$$R_1 \equiv \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) \frac{V(q^2)}{A_1(q^2)} \quad,$$
$$R_2 \equiv \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) \frac{A_2(q^2)}{A_1(q^2)} \quad, \tag{28}$$

where $V(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ denote vector and axial vector form factors respectively. Then we can write the relations among form factors and the Isgur-Wise function as

$$A_1(q^2) = \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) R^{*-1}\xi(q^2) \quad,$$
$$A_2(q^2) = R_2 R^{*-1}\xi(q^2) \quad,$$
$$V(q^2) = R_1 R^{*-1}\xi(q^2) \quad. \tag{29}$$

Note that the Isgur-Wise function $\xi(v \cdot v')$ in these expressions contains full $1/m_Q$ corrections and it corresponds to $\hat{\xi}(v \cdot v')$ in the Ref. [8], which is normalized at zero recoil up to the second order corrections $\hat{\xi}(1) = 1 + \delta_{1/m_Q}$. And R_1 and R_2 become to be unity in the heavy quark limit, and Neubert's estimates of the $1/m_Q$ corrections [1] give

$$R_1 \approx 1.3 \quad, \qquad R_2 \approx 0.8 \quad, \tag{30}$$

which are model dependent. Recently CLEO [10] obtained the values of the parameters R_1, R_2 by fitting them with the slope parameter of the Isgur-Wise function ρ^2. Since R_1 and R_2 have very weak q^2 dependence, in the CLEO analysis they approximately obtained the q^2 independent values,

$$R_1 = 1.30 \pm 0.36 \pm 0.16 \quad,$$
$$R_2 = 0.64 \pm 0.26 \pm 0.12 \quad. \tag{31}$$

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Hereafter we also take them to be constants for simplicity.

In our model the parameters N_1, N_2 and N_3 are represented in terms of R_1 and R_2 as follows,

$$
\begin{aligned}
N_1 &= \frac{N/2}{1 - 2q^2/(m_B + m_{D^*})^2} \left[-\frac{q^2}{(m_B + m_{D^*})^2} \cdot 2R_1^2 \right. \\
&\quad \left. + \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) \left(\frac{(1 + R_2)^2}{2} + \frac{m_B^2 + q^2}{2m_{D^*}^2}(1 - R_2)^2 + \frac{2m_B m_{D^*} - q^2}{2m_{D^*}^2}(1 - R_2^2)\right) \right] , \\
N_2 &= \frac{N}{2} \left[(1 + R_1^2) + \frac{2m_B m_{D^*}}{m_B^2 + m_{D^*}^2 - q^2}(1 - R_1^2) \right] , \\
N_3 &= N R_1 .
\end{aligned}
\tag{32}
$$

In the heavy quark limit, we know that $R_1 = R_2 = 1$. Using the expression $R_i = 1 + \mathcal{O}(1/m_Q)$ we can separate the leading contributions and $1/m_Q$ corrections in N_i's:

$$
\begin{aligned}
N_1 &= \frac{N}{4}(1 + R_2)^2 + \mathcal{O}(1 - R_2) , \\
N_2 &= \frac{N}{2}(1 + R_1^2) + \mathcal{O}(1 - R_1) , \\
N_3 &= N R_1 .
\end{aligned}
\tag{33}
$$

When R_1, $R_2 \to 1$, we explicitly see that $N_1 = N_2 = N_3 \to N$.

Now substituting $\xi(v \cdot v')$ for $\mathcal{F}(q^2)$ with the Eq. (22), the hadronic tensor for the decay $\bar{B} \to D^* e \bar{\nu}$ is given by

$$
\begin{aligned}
H_{\mu\nu}(B \to D^*) = \mathcal{F}(q^2) &\left[(1 - \frac{2q^2}{(m_B + m_{D^*})^2}) N_1 (p_{B\,\mu} p_{D^*\,\nu} + p_{B\,\nu} p_{D^*\,\mu}) \right. \\
&\quad \left. - 2(1 - \frac{q^2}{(m_B + m_{D^*})^2})(N_2 g_{\mu\nu} p_B \cdot p_{D^*} - i N_3 \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D^*}^\beta) \right] .
\end{aligned}
\tag{34}
$$

The q^2 dependences of form factors are mainly determined by the function $\mathcal{F}(q^2)$, instead of the commonly used pole-dominance ansätze.

When we calculate the hadronic tensor within the HQET framework, we have generally some parameters parametrizing non-perturbative effects, which are obtained in model dependent ways. The slope parameter is such a characteristic parameter of the Isgur-Wise function, which represents the common behaviors of form factors. We calculated it, and

find that the value of the slope parameter is related to the parameters ϵ_b and ϵ_c in our approach. The HQET also contains the parameter $\lambda_a \sim -\langle k_Q^2 \rangle$ which is related to the kinetic energy of the heavy quark inside the heavy meson, and spin-symmetry breaking term $\lambda_2 = \frac{1}{4}(m_V^2 - m_P^2)$ corresponding to the mass splitting of pseudoscalar mesons and vector mesons. In our approach, we have two parameters R_1 and R_2 correspondingly, which are experimentally measurable. Also the values of ϵ_b and ϵ_c can be independently determined from the various methods experimentally and theoretically. We use the values for R_1 and R_2 from the CLEO fit results of Ref. [10], and for ϵ_b and ϵ_c from Ref. [9,11].

Finally, we obtain the decay spectrum

$$
\frac{d\Gamma(\bar{B} \to D^* e\bar{\nu})}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^5} \mathcal{F}(q^2) \left((m_B^2 - m_{D^*}^2 + q^2)^2 - 4m_B^2 q^2 \right)^{1/2}
$$
$$
\times \left[m_B^2 W_1(q^2) \left((m_B^2 - m_{D^*}^2 + q^2)^2 - m_B^2 q^2 \right) \right.
$$
$$
\left. + \frac{3}{2} m_B^2 W_2(q^2) (m_B^2 - m_{D^*}^2 + q^2) + 3m_B^2 W_3(q^2) \right] \quad , \tag{35}
$$

where

$$
W_1(q^2) = -N_1 \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) \quad ,
$$
$$
W_2(q^2) = N_1 (m_B^2 - m_{D^*}^2 + q^2) \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) - 2N_3 q^2 \left(1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) \quad ,
$$
$$
W_3(q^2) = -N_1 m_B^2 q^2 \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right)
$$
$$
+ N_3 q^2 (m_B^2 - m_{D^*}^2 + q^2) \left(1 - \frac{q^2}{(m_B + m_{D^*})^2} \right)
$$
$$
+ N_2 q^2 (m_B^2 + m_{D^*}^2 - q^2) \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) \quad , \tag{36}
$$

and $\mathcal{F}(q^2)$ is defined in (21). The result is plotted in Fig. 2 (b), also compared with the CLEO data [10]. The thick solid line is our model prediction with the parameters (ϵ_b =0.004, ϵ_c =0.04), the thin solid line the Wirbel *et. al.* model prediction [6], and the dotted line the Körner *et. al.* model prediction [7].

III. DISCUSSIONS AND CONCLUSIONS

All form factors show the same behavior for varying q^2, which is described by the Isgur-Wise function of the HQET, which represents the common non-perturbative dynamics of weak decays of heavy mesons. Ever since Fakirov and Stech [12], the nearest pole dominance has been usually adopted as the dependence of common behaviors on q^2. In our approach, their q^2 dependences are **derived** from the kinematic relations of b- and c-quark. When b-quark decays to c-quark, the momentum transfer to leptonic sector is equal to the difference between b-quark momentum and c-quark momentum in the parton picture. The b- and c-quark momenta within the B and D mesons have some specific distributions. For given momentum transfer q^2, there exist possible configurations of b- and c-quark momentum pairs $(p_b, \ p_c)$, and each pair is appropriately weighted with the momentum distributions of the quarks. Our $\mathcal{F}(q^2)$ function in (21) measures the weighted transition amplitude for given q^2 in the parton picture; it is explicitly given by the overlap integral of distribution functions of initial and final state hadrons. This is common to all form factors, as explained in Section II.

As mentioned earlier, all non-perturbative strong interaction effects are considered through the distribution functions in our model, so $\xi(v \cdot v')$ obtained from Eq. (22) corresponds to the hadronic form factor $\hat{\xi}(v \cdot v')$ defined in the Ref. [8], including $1/m_b$ corrections rather than the lowest order Isgur-Wise function. And the slope parameter of our results in Table 1 also corresponds to $\hat{\rho}^2$ related to $\hat{\xi}(v \cdot v')$. We obtain the values of the slope parameter ρ within the parton model framework, as in Table 1,

$$\rho^2 = 0.582 - 0.896 \ \ ,$$

which are compatible with the Neubert's prediction [8],

$$\hat{\rho}^2 \simeq \rho^2 \pm 0.2 = 0.7 \pm 0.2 \ \ .$$

Our result is rather smaller than the predictions of other models,

$$\rho^2 = 1.29 \pm 0.28 \qquad [13] \quad,$$

$$\rho^2 = 0.99 \pm 0.04 \qquad [14] \quad,$$

but it is consistent with the average value measured by experiments [15],

$$\hat{\rho}^2 = 0.87 \pm 0.12 \quad.$$

In calculating the numerical values, we still have two free parameters ϵ_b and ϵ_c of Eq. (6), *i.e.* of the heavy quark fragmentation functions. Their values can be determined independently from the various experimental and theoretical methods[1]. For the parton model to be consistent with the HQET, we require that with the fixed value of the parameter ϵ_Q, all the appropriate results of the parton model approach agree with those of the HQET. In other words, the value of parameter ϵ_Q should be determined to give all the phenomenological results to coincide with those of the HQET. In this point of view, we have previously studied the parton model approach for inclusive semileptonic decays of B meson in the Ref. [16], and showed that the value $\epsilon_b \approx 0.004$ gives consistent results with those of the HQET. In this paper we use the value of ϵ_b as 0.004 or 0.006. The latter value is given by the experiments for the determination of the Peterson fragmentation function [11]. We find that our prediction of the slope parameter ρ^2 with the parameter $\epsilon_b = 0.004$ and $\epsilon_c = 0.04$ gives the best agreed value 0.705 with that of the HQET, $\hat{\rho}^2 = 0.7 \pm 0.2$. In this context, we conclude that our model with the parameter $\epsilon_b = 0.004$ gives consistent predictions with the HQET.

Phenomenologically, our model prediction on q^2 spectrum in the $B \to D^* e\nu$ decay shows a good agreement with the result of the CLEO [10], as shown in Fig. 2(b). If we let $f_Q(z) = \delta(1 - z)$ and $R_1 = R_2 = 1$ in our model, we can reproduce the lowest order results of the HQET, and obtain the similar plot with those of other models in Fig. 2(b). For the $B \to D e\nu$ decay, our results agrees with those of other models, as in Fig. 2(a).

---

[1]Theoretically, it is interesting to calculate $f_Q(z) = \delta(1-z) + \mathcal{O}(1/m_Q^2)$ within the HQET, which can reproduce the phenomenological predictions.

Finally we obtain the ratio of integrated total widths $\Gamma(B \to D^*)/\Gamma(B \to D) \approx 2.66$, which agrees with the experimental results [17]. It may be a phenomenological support of our model because this quantity is independent of the CKM elements $|V_{cb}|$, which has uncertainties in determining its value yet. The perturbative QCD corrections can be factorized in the decay width calculation [8,18], which does not affect the ratio $\Gamma(B \to D^*)/\Gamma(B \to D)$.

To summarize, we developed the new parton model approach for exclusive semileptonic decays of B-meson by extending the inclusive parton model, and by combining with the results of the HQET, motivated by Drell-Yan process. Without the nearest pole dominance ansätze, we derived the dependences of hadronic form factors on q^2. We also calculated numerically the slope of the Isgur-Wise function, which is consistent with the experimental results.

ACKNOWLEDGMENTS

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TABLES

TABLE I. The normalization constant N and the slope parameter ρ are shown with several values of the parameters ϵ_b, ϵ_c.

	$\epsilon_b =$ 0.004				$\epsilon_b =$ 0.006			
	$\epsilon_c = 0.04$	0.06	0.08	0.1	0.04	0.06	0.08	0.1
N	0.938	1.068	1.190	1.306	1.105	1.223	1.338	1.449
ρ^2	0.705	0.646	0.609	0.582	0.896	0.844	0.810	0.785

FIGURES

FIG. 1.　(a) The diagram of Drell-Yan Process. (b) The schematic diagram of semileptonic exclusive decay of B-meson in the parton model.

FIG. 2.　(a) q^2 spectrum in the $B \rightarrow De\nu$ decays. The solid line is our model prediction and the dotted line the Wirbel *et. al.* model prediction [6]. We used the values of parameters, ($\epsilon_b =$0.004, $\epsilon_c =$0.04). (b) q^2 spectrum in the $B \rightarrow D^*e\nu$ decays. The thick solid line is our model prediction and the thin solid line the Wirbel *et. al.* model prediction [6] and the dotted line the Körner *el. al.* model prediction [7]. We used the values of parameters, ($\epsilon_b =$0.004, $\epsilon_c =$0.04). The data are quoted from Ref. [10].
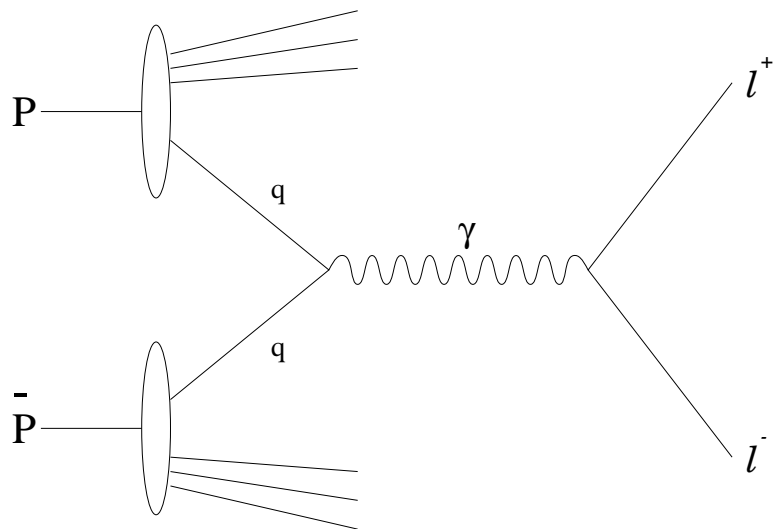
P

$\bar{P}$

q

q

γ

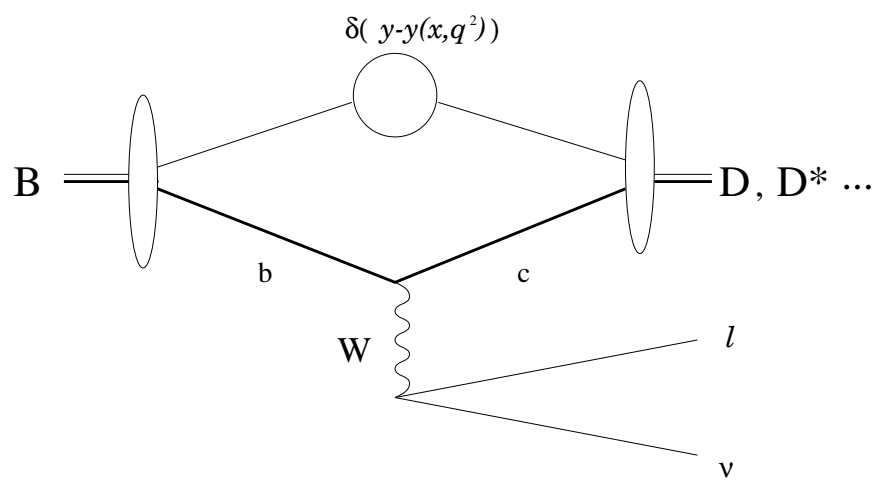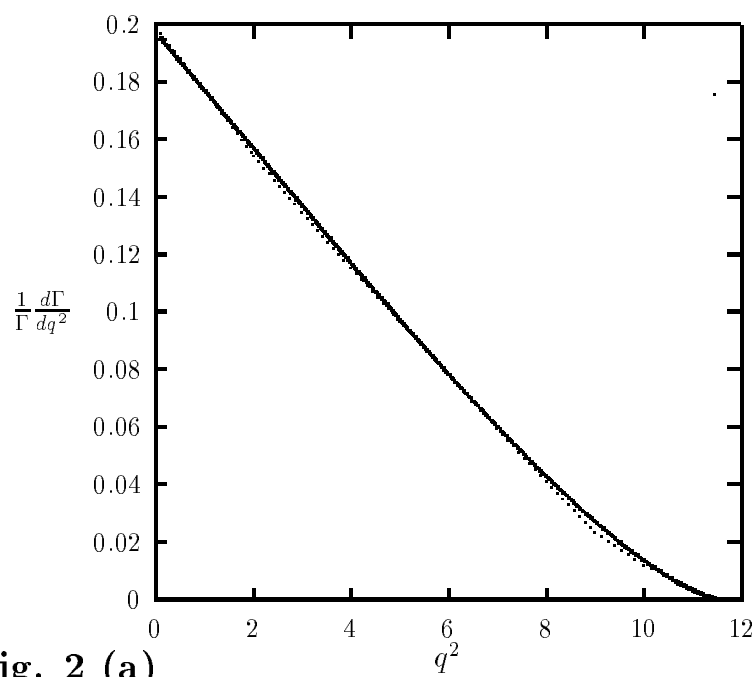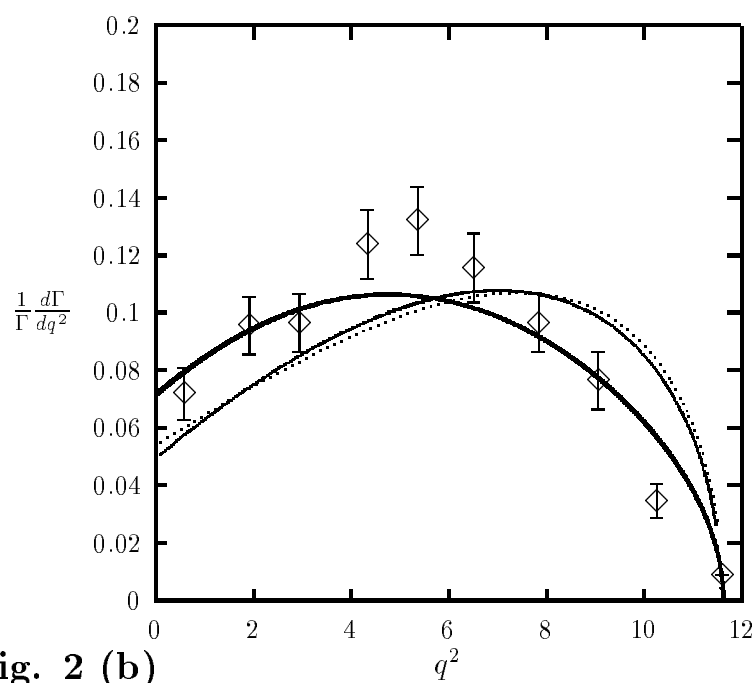l^+

l^-

Fig. 1 (a)

$\delta(\ y-y(x,q^2)\ )$

B

b

W

c

D, D* ...

l

ν

Fig. 1 (b)

Fig. 2 (a)

Fig. 2 (b)